GCSE Maths – Geometry and Measures

Vocabulary and Notation

Notes

WORKSHEET

Vocabulary and Notation

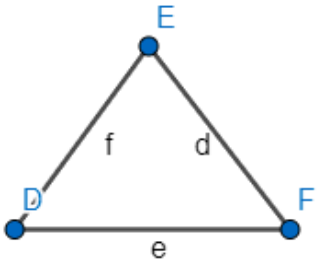
Geometry is the branch of mathematics that deals with the properties, measures and relationships of points, lines, shapes and solids.

Points and Lines

A **point** is a dot or cross on a diagram that can be labelled. Points can be connected by either straight or curved **lines**, and linked together to form **shapes**. Points are often labelled using upper-case letters (A, B, C), and lines are often labelled using lower-case letters (a, b, c). In a shape, the line **opposite** a point is labelled using the same letter.

The point A.

The points B and C are connected by the line l.



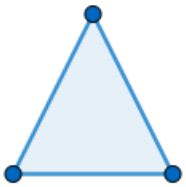
The points D, E and F connected by lines d, e and f form a shape.

Shapes

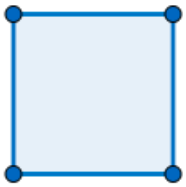
Lines and points form shapes. Shapes can have any number of points and lines.

2-D Shapes

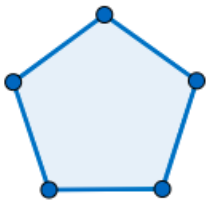
2-Dimensional shapes exist in two dimensions – **width** and **height**. They exist on a single plane (a flat, two-dimensional surface with no thickness that extends in every direction). We can imagine a plane as an endless piece of paper. 2-D shapes are also called **polygons**.



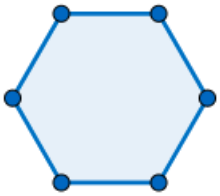
3 sides: triangle



4 sides: square



5 sides: pentagon



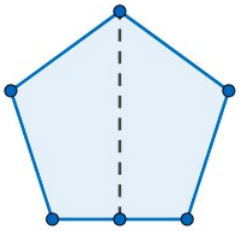
6 sides: hexagon

Polygons have a number of lines (or sides) that can be of any length. A polygon is **regular** if its **sides** are all the **same length**, and its **interior angles** are all the **same**.

Symmetries

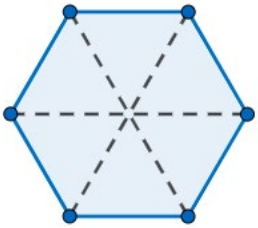
Shapes can have **reflectional** or **rotational** symmetries.

Reflectional symmetry is easy to see because one side of a shape is the same (or a reflection of) the other half. The two sides are split by a lime of symmetry. The line of symmetry can be in any direction, as long as the shape is mirrored on both sides.



A pentagon with one line of symmetry.

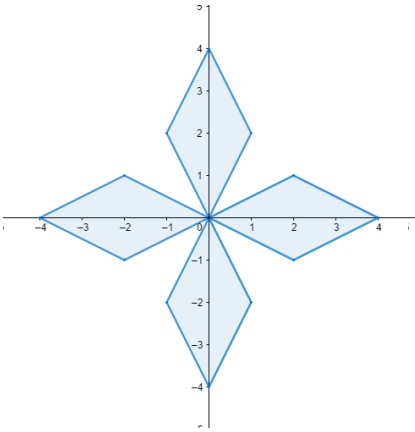
Can you find the other 4 lines of symmetry?



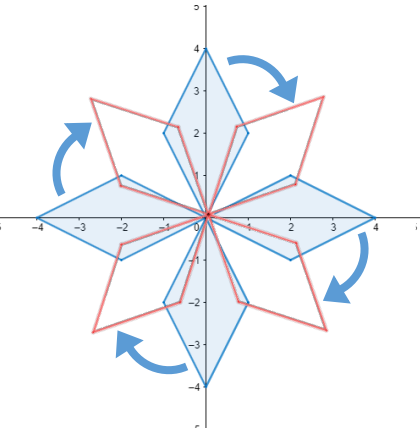
A hexagon with three lines of symmetry.

Can you find the other 5 lines of symmetry?

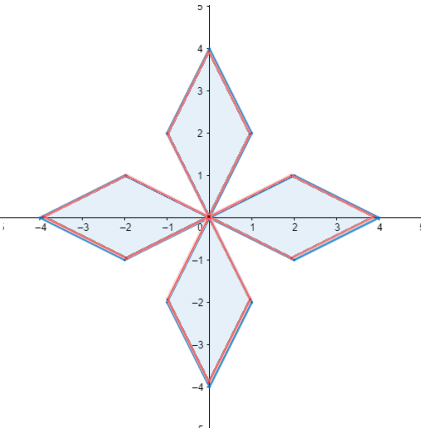
A shape with **rotational symmetry** will still look the same after it has been rotated (turned around a point) by less than one full turn. The number of times it matches up with the original shape during the rotation is called the **order**.



A polygon on an axis is rotated in the clockwise direction.



It is turned around the centre point until the rotated shape matches up with the original lines.

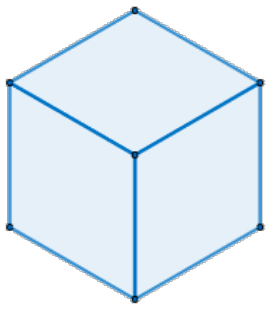


This shape has order 4, because it will match up with the original shape 4 times in one full turn.

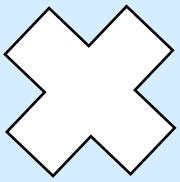
3-D Shapes

3-Dimensional shapes exist in three dimensions – **width, height** and **depth**.

The corners on a 3D shape, which can be thought of as points, are called **vertices**. The flat surfaces are called **faces**, and the connections between vertices where two faces meet are called **edges**.

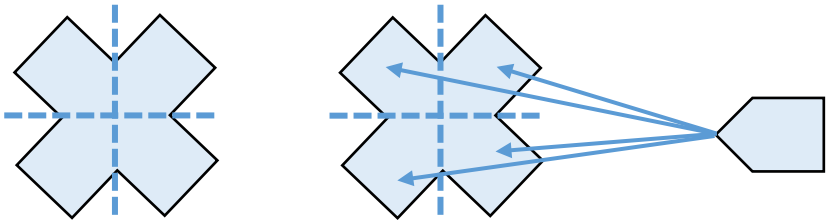


Example: Draw two lines of symmetry on the shape below:

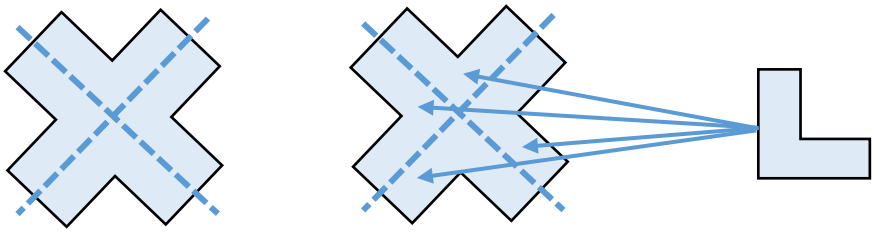


The question is asking you to draw two lines of symmetry – this means you need to find two 'folds' or 'cuts' through the shape that will create two more identical but mirrored shapes.

- *One solution is to cut the shape vertically and horizontally, so that each segment is the shape of an arrow, as shown below on the right:*



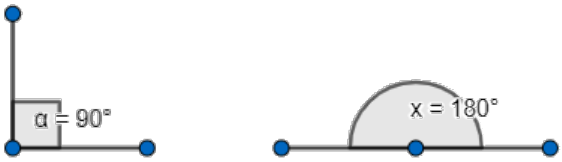
- *Another solution is the cut the shape diagonally, so that each segment is L-shaped:*



Angles

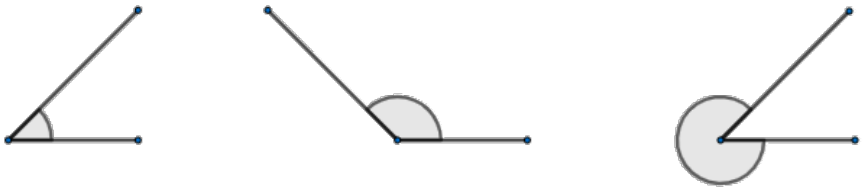
An angle is the amount of a turn. It is measured in degrees (°). There are $360°$ in a full turn (a circle), $180°$ in a half turn and $90°$ in a quarter of a turn. A $90°$ quarter turn is also called a **right angle**.

Right angles are marked with a square between the two lines. Any other angle is marked with a sector of a circle between the two lines.



Types of angles

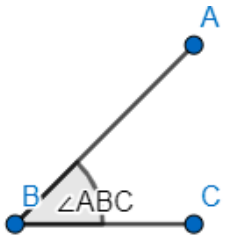
Angles less than 90° are **acute**. Angles more than 90° (a right angle) and less than 180° (a straight line) are **obtuse**. Angles more than 180° and less than 360° are **reflex**.



Acute angle | Obtuse angle | Reflex angle

Naming angles

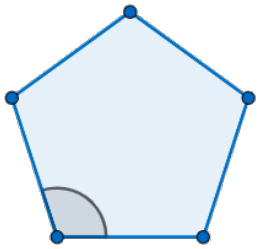
Angles can be named using a **lower-case letter** – such as x or a – or by describing the **location** of the angle using the points around it. For example, an angle between the points A, B and C, around the point B, is labelled $\angle \boldsymbol{ABC}$.

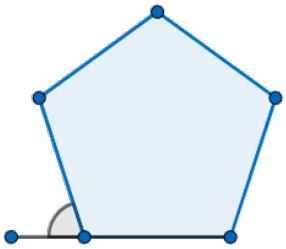


Angles in a shape

There are two types of angles in a shape - interior angles, and exterior angles:

- **Interior** angles are on the **inside** of the shape at the vertex.
- **Exterior** angles are measured outside of the shape from the **extension of one side** to the **next side.**


Interior angle


Exterior angle

The sum of interior angles can be found by adding the interior angle at each vertex.

Regular Shapes

Shapes are regular if all their **sides are the same length**, and all **interior angles are equal**. The sum of interior angles for a shape is based on the **number of sides** that the shape has.

Number of Sides	Name of Shape	Sum of Interior Angles	Value of Each Interior Angle in a Regular Polygon
3	Triangle	180°	60°
4	Quadrilateral	360°	90°
5	Pentagon	540°	108°
6	Hexagon	720°	120°
10	Decagon	1140°	144°
100	Hectogon	17640°	176.4°

The table shows a **pattern** in the sums of interior angles. For a shape with $\boldsymbol{n}$ sides, the sum of interior angles follows the following formula:

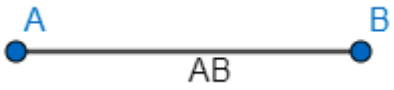
$$Sum\ of\ interior\ angles\ of\ n-sided\ shape = \mathbf{180°} \times (\boldsymbol{n}-\mathbf{2})$$

To find the value of **each interior angle in a regular polygon**, divide this value by the number of sides, $\boldsymbol{n}$:

$$Value\ of\ each\ interior\ angle\ in\ an\ n-sided\ shape = \frac{\mathbf{180°} \times (\boldsymbol{n}-\mathbf{2})}{\boldsymbol{n}}$$

Lines

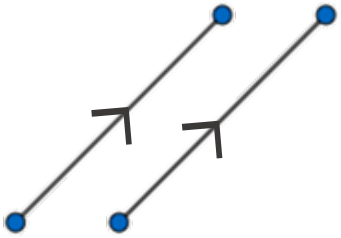
A line **connects two points**, and can be referred to using those points. For example, the line that connects point A and point B could be referred to as the **line AB**.



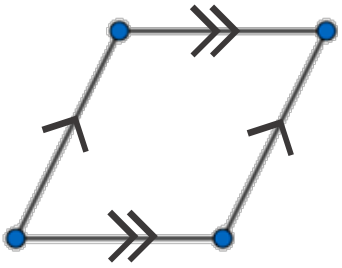
Parallel lines

Lines that are parallel are a **fixed distance apart** at all points, and will **never touch** regardless of how long they are extended.

To show that two lines are parallel, they are **marked with arrow signs** (>) pointing in the same direction.



A pair of parallel lines is marked with arrows in the direction of the lines.

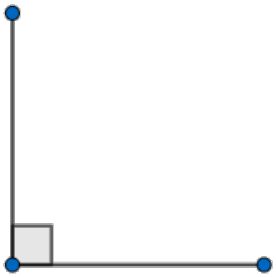


If there are multiple sets of parallel lines, mark each pair with matching numbers of arrows.

(Higher Only) Lines that are parallel also have the **same gradient**.

Perpendicular lines

Lines that are perpendicular meet at a **90° angle** (a right angle). They are marked with a right-angle sign which looks like a box (□).



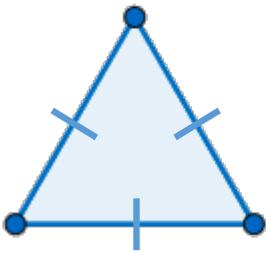
The sides of regular **quadrilaterals** (4-sided shapes) are perpendicular.

(Higher Only) Two lines are **perpendicular** if the product of their gradients is -1.

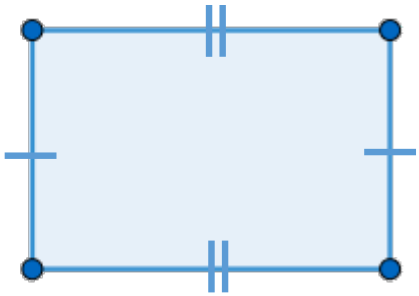
Lines in polygons

If the lines in a polygon are of **equal length**, they are marked with a small bar (–).

If all the lines in a shape are of **equal length**, and all interior **angles are equal**, then the polygon is **regular**.



A pair of equal lines is marked with bars in the centre of the lines.



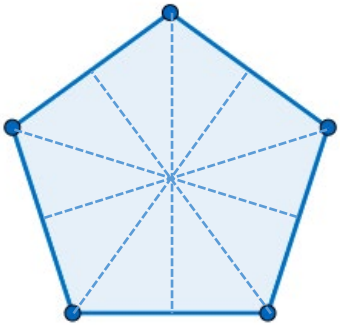
If there are multiple sets of equal lines, mark each pair with matching numbers of bars.

Example: A shape has five lines of symmetry. All the sides are of equal length. Draw and name the shape.

If a shape has 5 lines of symmetry, it is likely to have n sides where n is a multiple of 5.

Draw a shape with 5 sides. Since, all sides are the same length, the shape must be regular.

*The shape has 5 sides, so it is called a **pentagon**.*

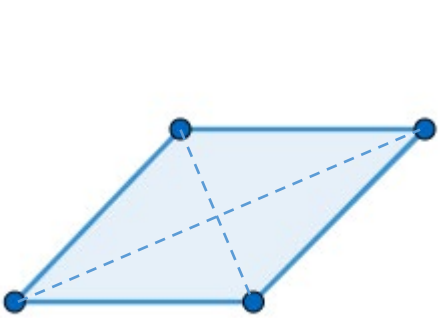


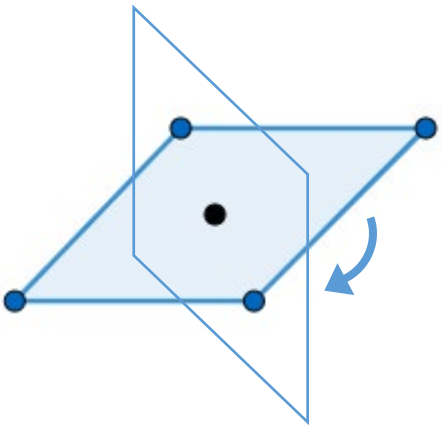
Example: Draw a shape with two lines of symmetry and rotational symmetry of order 2.

The shape must have two lines of symmetry (mirror lines) so that it is the same on all sides.

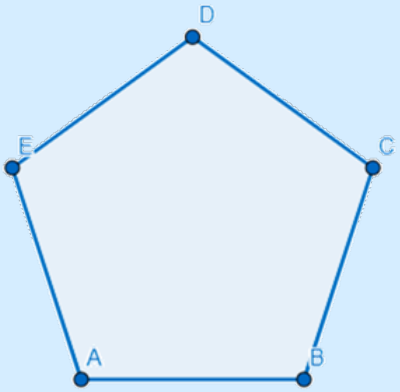
It must also have rotational symmetry of order 2 – so when it is rotated, it should match up with the original shape twice in one full rotation.

*The shape could be a **rhombus**.*



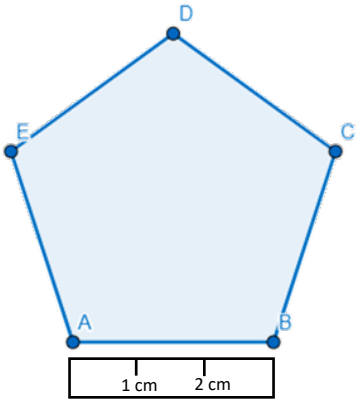


Example: In polygon ABCDE, draw a line through the midpoint of AB that is perpendicular to AB.



ABCDE is a regular pentagon. This means that the line from the midpoint of AB that is perpendicular to AB will pass through the point D.

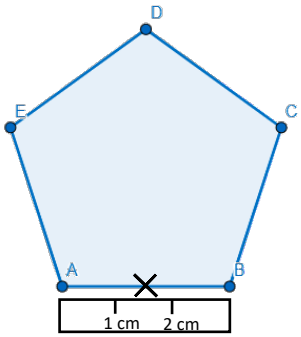
1. Measure the length of AB using a ruler. Divide the length by 2 to find the distance from A to the midpoint. Mark the midpoint on the diagram.



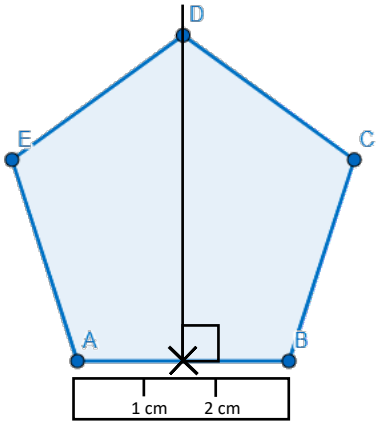
The length of AB is 3 cm.

$$3 \div 2 = 1.5$$

Therefore, the midpoint of AB is 1.5 cm *from A.*



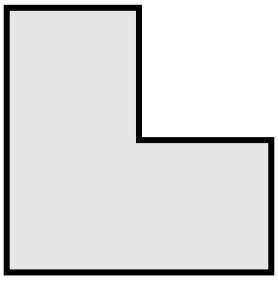
2. Draw a line from the midpoint of AB through point D. Mark the line as perpendicular to line AB.



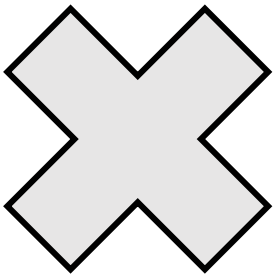
Vocabulary and Notation – Practice Questions

1. Draw the given number of lines of symmetry on the shapes below.

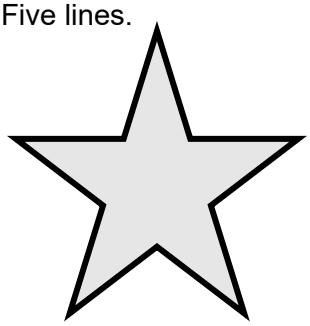
   a) One line.

   b) Two lines.

   c) Five lines.

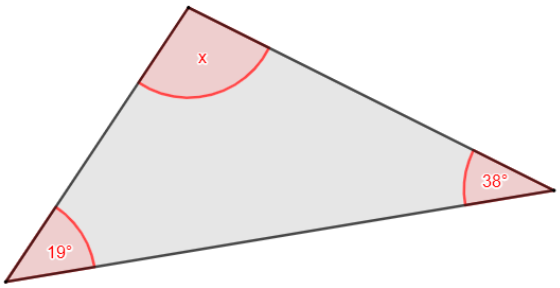
   

2. Draw examples of the following angles:

   a) An acute angle.
   b) A reflex angle.
   c) A half-turn.

3. The diagram below shows a triangle with three angles marked.

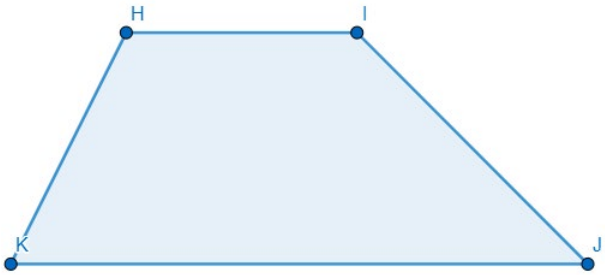
   Daniel says, "The size of angle x is 76°."

   Is Daniel correct? Give a reason for your answer.

4. In the trapezium HIJK:

   a) Mark the parallel lines.
   b) Draw a line through the midpoint of HI that is perpendicular to KJ. Is this a line of symmetry? Explain your answer.

Worked solutions for the practice questions can be found amongst the worked solutions for the corresponding worksheet file.